Ex- If $\alpha, \beta, \gamma, \delta$ be the roots of the biquadratic equation $x^4 + px^3 + qx^2 + rx + s = 0$. Find in terms of the coefficients, the value of the following symmetric functions:
(i) $\Sigma\alpha^2$ (ii) $\Sigma\alpha^2\beta$. (iii) $\Sigma\alpha^2\beta\gamma$ (iv) $\Sigma\alpha^3$
(v) $\Sigma\alpha^2\beta^2$ (vi) $\Sigma(\alpha-\beta)^2(\gamma-\delta)^2$ (vii) $\Sigma(\gamma\alpha+\beta\delta)(\alpha\beta+\gamma\delta)$

Soln:-

Since $\alpha, \beta, \gamma, \delta$ are the roots of equation
$$x^4 + px^3 + qx^2 + rx + s = 0 \quad \text{—(1)}$$

we have,
$$\Sigma\alpha = -p, \; \Sigma\alpha\beta = q, \; \Sigma\alpha\beta\gamma = -r \text{ and } \alpha\beta\gamma\delta = s$$

(i) $\Sigma\alpha^2 = (\Sigma\alpha)^2 - 2\Sigma\alpha\beta$
$= (-p)^2 - 2q = p^2 - 2q$

(ii) we have,
$\Sigma\alpha \cdot \Sigma\alpha\beta = (\alpha+\beta+\gamma+\delta)(\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta)$
$\Sigma\alpha \cdot \Sigma\alpha\beta = \Sigma\alpha^2\beta + 3\Sigma\alpha\beta\gamma$

$\Rightarrow \Sigma\alpha^2\beta = \Sigma\alpha \cdot \Sigma\alpha\beta - 3\Sigma\alpha\beta\gamma$
$= (-p) \cdot q - 3 \cdot (-r) = 3r - pq$

(iii) we have
$\Sigma\alpha \cdot \Sigma\alpha\beta\gamma = (\alpha+\beta+\gamma)(\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta)$
$= \Sigma\alpha^2\beta\gamma + 4\alpha\beta\gamma\delta$

$\therefore \Sigma\alpha^2\beta\gamma = (\Sigma\alpha \cdot \Sigma\alpha\beta\gamma) - 4\alpha\beta\gamma\delta$
$= (-p)\cdot(-r) - 4s$
$= pr - 4s$ Ans

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(iv) we have, $\Sigma\alpha \cdot \Sigma\alpha^2 = (\alpha+\beta+\gamma+\delta)(\alpha^2+\beta^2+\gamma^2+\delta^2)$

$$= \Sigma\alpha^3 + \Sigma\alpha^2\beta$$

$$\therefore \Sigma\alpha^3 = \Sigma\alpha \cdot \Sigma\alpha^2 - \Sigma\alpha^2\beta$$

$$= (-p)\cdot(p^2-2q) - (3r-pq)$$

$$= -p^3 + 2pq - 3r + pq$$

$$= 3pq - p^3 - 3r$$

(v) we have, $(\Sigma\alpha\beta)^2 = (\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta)^2$

$$= \Sigma\alpha^2\beta^2 + 2\Sigma\alpha^2\beta\gamma + 6\alpha\beta\gamma\delta$$

$$\therefore \Sigma\alpha^2\beta^2 = (\Sigma\alpha\beta)^2 - 2\Sigma\alpha^2\beta\gamma - 6\alpha\beta\gamma\delta$$

$$= q^2 - 2(pr-4s) - 6s \quad \{\text{from (iii)}\}$$

$$= q^2 - 2pr + 8s - 6s$$

$$= q^2 - 2pr + 2s$$

(vi) $\Sigma(\alpha-\beta)^2(\gamma-\delta)^2 = \Sigma(\alpha^2+\beta^2-2\alpha\beta)(\gamma^2+\delta^2-2\gamma\delta)$

$$= 2\Sigma\alpha^2\beta^2 - 2\Sigma\alpha^2\beta\gamma + 12\alpha\beta\gamma\delta$$

$$= 2(q^2-2pr+2s) - 2(pr-4s) + 12s$$

$\{\text{from (v) \& (iii)}\}$

$$= 2q^2 - 4pr + 4s - 2pr + 8s + 12s$$

$$= 2q^2 - 6pr + 24s$$

(vii) $\Sigma(\gamma\alpha+\beta\delta)(\alpha\beta+\gamma\delta) = \Sigma\alpha^2\beta\gamma = pr - 4s \quad \{\text{from (iii)}\}$